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TRANSVERSE VIBRATIONS OF A SIMPLY SUPPORTED PLATE OF GENERALIZED ANISOTROPY WITH AN OBLIQUE CUT-OUT

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1. INTRODUCTION

Transverse vibrations of thin, elastic plates of different shapes with cut-outs have been studied by several researchers. Several studies have appeared in the case of circular, square and rectangular cut-outs with free edges [1–3]. A recent contribution on the subject matter can be found in reference [4].

The present paper deals with the approximate analytical determination of the lower frequencies of the transverse vibration of rectangular plates of generalized anisotropy and with a free oblique edge.

It is important to point out that cut-outs of the type shown in Figure 1 arise very commonly in engineering application in everyday practice: from civil engineering slabs to electronic chassis, and plants used in naval, aeronautics and nuclear engineering applications.

For the problem at hand, an approximate analytical solution is obtained expressing the displacement amplitude in terms of a double Fourier series which identically satisfies the boundary conditions of the original, rectangular plate. The frequency determinant is generated using the classical Rayleigh–Ritz method by deducting the subsidiary functional, corresponding to the cut-out, from the energy functional corresponding to the original plate.

2. APPROXIMATE ANALYTICAL SOLUTION

The displacement amplitude is approximated using the truncated double Fourier series

$$W \cong W_a = \sum_{1}^{N} \sum_{1}^{M} b_{nm} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}. \tag{1}$$

According to the classical Rayleigh–Ritz method, one generates the following functional from the maximum strain and kinetic energies of the plate of Figure 1:

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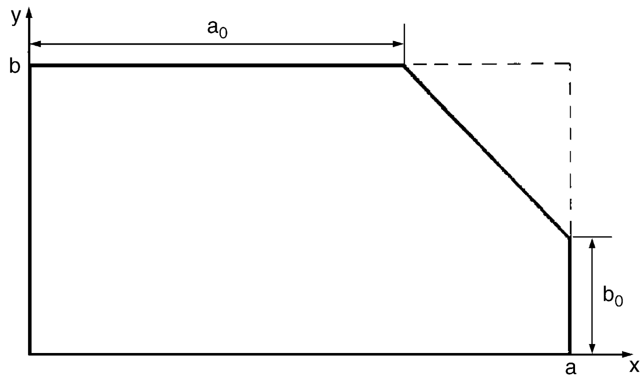

Figure 1. Simply supported rectangular plate of generalized anisotropy with a free oblique edge considered in the present study.

$$J[W] = \frac{1}{2}\iint \left\{ D_{11}\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_{12}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + D_{22}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2 + 4\left[D_{16}\left(\frac{\partial^2 W}{\partial x^2}\right) + D_{26}\left(\frac{\partial^2 W}{\partial y^2}\right)\right]\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)\right\} \mathrm{d}x\,\mathrm{d}y - \frac{\rho\omega^2 h}{2}\iint W^2\,\mathrm{d}x\,\mathrm{d}y, \quad (2)$$

where the well-established Lekhnitskii notation [5] has been used.

As has been stated before, the integrals in expression (2) extend over the actual area of the cut-out plate under study. Then, one gets the frequency determinant whose roots are the frequency coefficients $\Omega_i = \sqrt{\rho h/D_{11}}\omega_i a^2$ in a straightforward albeit rather lengthy analytical procedure.

3. NUMERICAL RESULTS

All calculations were performed taking $D_{12}/D_{11} = 0{\cdot}3$; $D_{22}/D_{11} = 0{\cdot}5$; $D_{66}/D_{11} = 0{\cdot}5$; $D_{16}/D_{11} = D_{26}/D_{11} = 1/3$ and $N = M = 20$ in equation (1).

Tables 1–3 show numerical results for simply supported rectangular plates of three different aspect ratios, namely $b/a = 2/3$, 1 and 3/2. For simplicity, the cut-outs have been chosen to be of the same aspect ratio as the original whole plate.

The results quoted include values of the cut-out from $a_0/a = b_0/b = 1$ down to $a_0/a = b_0/b = 0{\cdot}5$ in 0·05 steps. In a previous paper [4], dealing with isotropic and orthotropic plates, analytical results for $a_0/a = b_0/b = 0{\cdot}5 - 0{\cdot}7$ appeared to be rather high upper bounds. In this work, we have used what we believe is an improved algorithm and we are confident that the quoted results will prove to be of a greater degree of accuracy. However, numerical instabilities arise for $_3$ in the case of Table 1 for $a_0/a = b_0/b = 0{\cdot}75$, 0·70 and 0·65.

As can be ascertained from the tables, all quoted frequency coefficients experience a decrement in their values as the free oblique edge is being introduced into the plate. But the variation of both the fundamental frequency coefficient and Ω_3 seem to be slower than that of Ω_2 and Ω_4. This same behavior was encountered in the case of isotropic and orthotropic plates [4].

TABLE 1

Frequency coefficients of an anisotropic rectangular plate with a free, oblique edge (b/a=2/3)

$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
1·00	28·214	56·003	79·769	97·917
0·95	28·160	55·746	79·722	97·574
0·90	27·816	55·550	79·253	97·139
0·85	27·316	55·386	78·871	96·727
0·80	26·886	55·082	78·800	96·219
0·75	26·644	54·402	78·902	95·639
0·70	26·489	53·214	79·011	94·952
0·65	26·396	51·457	78·949	93·866
0·60	26·133	49·105	78·402	92·396
0·55	26·047	46·339	76·824	91·835
0·50	25·719	43·191	73·941	91·062

TABLE 2

Frequency coefficients of an anisotropic square plate with a free, oblique edge (b/a=1)

$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
1·00	18·839	38·003	52·121	65·878
0·95	18·808	37·910	52·066	65·542
0·90	18·605	37·816	51·753	65·097
0·85	18·292	37·714	51·503	64·566
0·80	18·019	37·503	51·558	64·035
0·75	17·863	37·136	50·871	63·285
0·70	17·792	36·683	50·191	62·347
0·65	17·761	35·958	49·378	61·542
0·60	17·746	34·499	48·807	60·863
0·55	17·699	33·160	48·113	60·433
0·50	17·550	32·035	47·652	60·292

TABLE 3

Frequency coefficients of an anisotropic rectangular plate with a free, oblique edge (b/a=3/2)

$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
1·00	14·199	24·332	40·699	44·550
0·95	14·175	24·308	40·574	44·503
0·90	14·050	24·222	40·394	44·339
0·85	13·855	24·113	40·183	44·222
0·80	13·675	23·980	39·988	44·008
0·75	13·574	23·639	39·699	43·750
0·70	13·442	23·053	39·253	43·408
0·65	13·342	22·760	38·816	43·057
0·60	13·242	22·380	38·402	42·827
0·55	13·127	22·044	37·785	42·558
0·50	12·941	21·258	36·800	42·322

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